The Effect of Nonlinear Heat Conduction on the Pressure-Coupled Response of Solid Propellants

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Nomenclature

a = differential equation solution parameter
c_p = condensed-phase specific heat
k = temperature sensitivity parameter, $(T_s^o - T_o)(\partial \ell n r^o / \partial T_o)_p$
n = burn rate exponent, $(\partial \ell n r^o / \partial \ell n p)_{T_o}$
p = pressure
q = heat flux vector
r = burn rate
R_p = pressure-coupled response
t = time
T = temperature
z = root of characteristic equation
δ = pressure-coupled response parameter, $\nu n - \mu k$
ζ = relaxation time parameter, $\tau \rho c_p r^{o2} / \lambda$
λ = condensed-phase thermal conductivity
μ = surface temperature pressure exponent parameter $[1/(T_s^o - T_o)]\,[\partial T_s^o / \partial \ell n p]_{T_o}$
ν = surface temperature sensitivity parameter, $(\partial T_s^o / \partial T_o)_p$
ρ = condensed-phase density
τ = relaxation time
ω = frequency
Ω = nondimensional frequency, $\lambda\omega / r^{o2} \rho c_p$

Superscript

o = steady state

Subscripts

i = imaginary part
o = initial condition
r = real part

Abstract

A ZELDOVICH-NOVOZHILOV model for the pressure-coupled response of solid propellants was developed with heat conduction in the condensed phase governed by a relaxation time, non-Fourier relation. Results indicate that, although the relaxation time appears in a frequency-squared relaxation time product combination, the non-Fourier model has negligible effect on the response in the frequency domain of interest to combustion instability for reasonable values of relaxation time. For large values of this parameter, the resonance is shifted to lower frequencies, reduced in magnitude, and narrowed.

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Contents

In all models for the pressure-coupled response of solid propellants, the solution of the condensed-phase energy equation incorporates the Fourier heat conduction law.[1,2] A known anomaly of this formulation is its characteristic infinite speed of thermal propagation, which is invalid in short transients and high heat flux processes.[3,4] These situations occur in the condensed phase at high frequencies, thereby suggesting that another formulation of the heat conduction

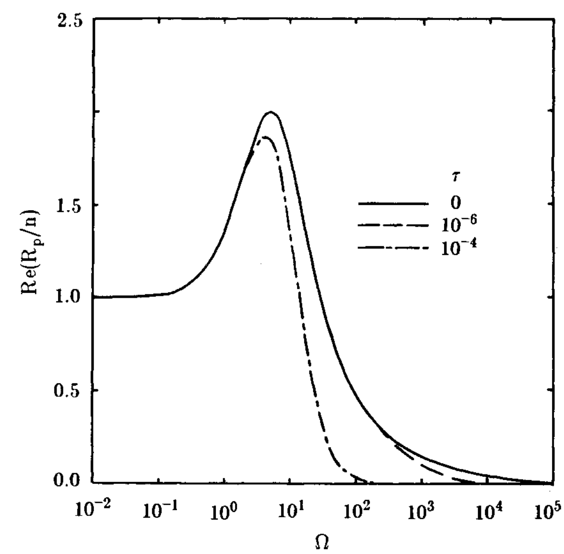

Fig. 1 Real component of R_p vs Ω.

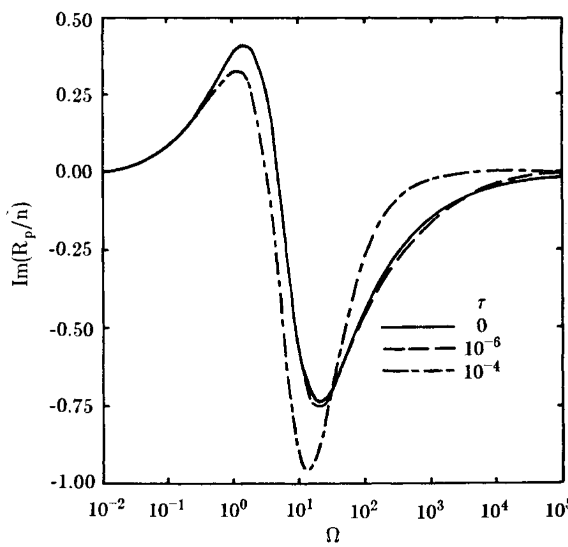

Fig. 2 Imaginary component of R_p vs Ω.

Table 1 Steady-state propellant properties

Activation energy	15000 cal/mole
Burn rate (6.9 MPa)	0.75 cm/s
Burn rate exponent	0.67
Initial propellant temperature	298 K
Surface temperature (6.9 MPa)	623 K
Temperature sensitivity	0.003 1/K
Thermal diffusivity	0.0013 cm^2/s

law, which accounts for the finite propagation speed, be employed. Models based on statistical mechanics and postulational bases are available and have the form[4]

$$q = -\lambda \nabla T - \tau \frac{\partial q}{\partial t} \tag{1}$$

where τ is a relaxation time. In this work, the above was mated to the Zeldovich-Novozhilov (ZN) nonsteady model[5] to produce the desired pressure-coupled response. The ZN model was employed because of its simplicity and independence from kinetic details. This is largely achieved through the assumption of a quasisteady reactive region.

The ZN methodology allows the pressure-coupled response function to be written in terms of steady-state parameters as[6]

$$R_p = \frac{\delta z + n}{1 - k + (\nu - ka) z} \tag{2}$$

The quantities a and z are determined by the solution of the condensed-phase energy equation, which depends on the non-Fourier heat conduction law and are found to be

$$z = \frac{((1 - 4\Omega^2\zeta - \Omega^2\zeta^2) + i(4\Omega + 2\Omega\zeta))^{1/2} - (1 - i\Omega\zeta)}{2} \tag{3}$$

and

$$a = \frac{-\zeta^2}{\Omega^2\zeta^2 + (1+\zeta)^2} + i\frac{\Omega^2\zeta^2 + 1 + \zeta}{\Omega^3\zeta^2 + \Omega(1+\zeta)^2} \tag{4}$$

Equation (3) can be further simplified by separating its real and imaginary parts to yield

$$z_r = \left[\frac{1}{4} - \Omega^2\zeta - \frac{\Omega^2\zeta^2}{4} + \left(\frac{\Omega^4\zeta^4}{16} + \frac{\Omega^4\zeta^3}{2} + \Omega^4\zeta^2 + \frac{\Omega^2\zeta^2}{8} + \frac{\Omega^2\zeta}{2} + \Omega^2 + \frac{1}{16}\right)^{1/2} \Big/ 2\right]^{1/2} - \frac{1}{2} \tag{5}$$

and

$$z_i = \frac{\Omega\zeta z_r + \Omega\zeta + \Omega}{2z_r + 1} \tag{6}$$

Inspection of the relations for z and a provides insight into non-Fourier heat conduction effects. First, as the relaxation time approaches zero, the solution approaches the classical ZN result.[5] Therefore, correct behavior is exhibited in this limit. Second, since the relaxation time is small $[0(10^{-10} - 10^{-14}\ \text{s})]$, only terms of the form $\Omega^2\zeta$ will be important. As $0(\zeta) \sim 10^{-7}$, significant effects will occur only when $0(\Omega) \geq 10^3$ and only z_r will be modified. The value $\Omega \sim 1$ corresponds roughly to the response function peak. Therefore, non-Fourier effects can be expected to be significant at frequencies three orders of magnitude beyond that. For typical conditions, this is the megahertz regime. Consequently, non-Fourier effects are negligible in the frequency domain of interest to combustion instability.

Figures 1 and 2 present results for the real and imaginary part of the pressure-coupled response for a hypothetical double-base propellant (Table 1 presents propellant properties.) These results validate the above deductions and show that, if relaxation effects were to occur, they would narrow and reduce the resonance and shift the peak to lower frequencies; however, the basic trends would remain unchanged.

Acknowledgments

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